A Study of the surface diffuseness of inter-nucleus potential with quasi-elastic scattering for the $^{32,34}_{16}\text{S} + ^{208}_{82}\text{Pb}$ reactions

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Abstract
Precise systematic studies on the surface diffuseness parameter of the nuclear potential for the heavy-ion reactions involving the systems have been achieved by using large-angle quasi-elastic scattering at deep sub-barrier energies close to the Coulomb barrier height. The single-channel (SC) and coupled-channels (CC) calculations have been carried out to elicit the diffuseness parameter of the nuclear potential. The chi square method $\chi^2$ has been used with a view to find the best fitted value of the diffuseness parameter in comparison with the experimental data. The surface diffuseness parameters have been elicited from the coupled-channels calculations with inert projectile and vibrational target are in complete agreement with the standard value which is (0.63 fm) while the single-channel calculations give to a certain extent larger values in the range from 0.64 fm to 0.65 fm.

Keywords
quasi-elastic scattering, Heavy-ion fusion reactions, deep sub-barrier energies, Coupled-channels calculations.

PACS number (s)
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1. Introduction

Knowing of the nucleus-nucleus interaction potential is the main component in the analysis of nuclear reactions [1, 2] and it has been played a crucial role [3] so as to describe nucleus-nucleus collisions [4]. The nucleus-nucleus potential is the reason in the interaction energy of colliding nuclei [2, 5, 6], it has been used to estimate the cross sections of various nuclear reactions [1, 2], moreover, in deformed nucleus interaction the nucleus-nucleus potential rely on the orientation angle of the deformed nucleus relative to the beam direction [7, 9]. We can define the nucleus-nucleus potential as the sum of the nuclear potential $V_{N(r)}$ which is less defined and the Coulomb potential $V_{C(r)}$ which is well-known [1, 4]. By the precise description of the Coulomb or Rutherford scattering [4, 10]. The barrier height of the nucleus-nucleus reaction rely on the ratio between the nuclear and Coulomb potentials, that work at teeny distances between the surfaces of reactant nuclei [5]. Consequently, the nucleus-nucleus potential is consist from Coulomb and nuclear parts, so that long range repulsion Coulomb potential acts between the protons in nuclei while the nuclear interaction between nucleons [5], the nuclear part is commonly expressed by the Woods-Saxon (WS) form [11], which is discriminated by the deepness $V_o$, radius $r_o$, and diffuseness $a$ parameters [12]. The fact that the WS form of a simple exponential had been exploited to research the surface characteristic of nuclear potential [13]. The WS potential has great importance in nuclear physics due to be considered reasonable potential [14]. The value of surface diffuseness parameter which was accepted, it is around 0.63 fm has been used for accounts of elastic and inelastic scattering, which are sensitive fundamentally to the surface region of the nuclear potential [15]. We can study the nuclear potential through quasi-elastic scattering or fusion experimental data [10].

Quasi-elastic scattering can be defined as sum of elastic scattering, inelastic scattering and transfer reaction [16, 19], it is very well equivalent of the fusion reaction [16, 19, 20], which is defined as a reaction where two discrete nuclei integrate together to form compound system [21, 22]. Fusion and Quasi-elastic scattering are both considered extensive operations and are complementary to each other [13, 23, 24]. As a result, these interactions are subject to the same potential and share the same information about the mechanism of interaction, and both are sensitive to the channel coupling Impacts (due to collective inelastic excitements of the colliding nuclei) at energies near the Coulomb barrier [19, 20]. Experimentally, the measurement of quasi-elastic scattering more easier than that of fusion interaction, particularly at deep sub-barrier energies [13, 20]. As well as note that the scattering operation is sensitive fundamentally to the surface area of the nuclear potential, whilst the fusion reaction is also comparatively sensitive to the internal fraction [3, 15].

The experimental measurement process to large-angle quasi-elastic scattering cross sections are more efficient and easier than the fusion cross sections [10]. That the perversion of the rate of the quasi-elastic to the Rutherford cross sections from unity at deep sub-barrier energies provides a clear way to set the account of the surface
diffuseness parameter in the nucleus-nucleus potential [13]. Consequently, can be defined the diffuseness parameter as a landing of the nuclear potential and thus directly impacts on the barrier width and the coupling strong points which to first order rely on the derivative of the potential [25, 26]. It is one-component parameters of the WS potential, which is known downhill nuclear potential in the tailpiece area of Coulomb barrier [27, 29].

Coupling channel model is an ideal tool to reproduce the experimental data at the same time for several processes, such as elastic, inelastic scattering, particle transfers and fusion within a unified framework [21, 30]. The inter-nuclear potential is the most important component in the coupled-channels calculations [30], such that the nuclear potential affect the width of the barrier and the coupling strengths [26]. The channel coupling is caused by coupling of the internal degrees of freedom which are included the transfer reactions and the collective vibrational and rotational motions with the relative motion of the colliding nuclei [10, 12, 18]. In nucleus-nucleus collisions at deep sub-barrier energies near the Coulomb barrier, observed that the effect of coupling channels can be neglected, because reflection probability is nearly unity at such energies, however, this analysis would be acceptable for the spherical nuclei collisions [10, 12, 15]. The use of coupling channels accounts does not play an important role in determining the best value for the diffuseness parameters at deep sub-barrier energies, but the essential purpose of employ these accounts is to achieve the effects of some calculation inputs on the resulting diffuseness parameters. The excitation states of the colliding nuclei play an important role to perform coupled-channels calculations [31].

K. Washiyama et al. [15] had been performed study on the surface characteristic of nucleus-nucleus potential in heavy-ion reactions using large-angle quasi-elastic scattering at energies much less the Coulomb barrier. Consequently, single-channel was suitable potential model to describe these energies. They had concluded that systems which involve deformed target require the diffuseness parameter between 0.8 fm and 1.1 fm, whilst spherical nuclei systems require the diffuseness parameter of around 0.60 fm.

K. Jassim et al. [4] have analyzed on the nuclear potential for heavy ion systems, namely $^{48}$Ti, $^{54}$Cr, and $^{64}$Ni + $^{208}$Pb systems by using large-angle quasi-elastic scattering at sub-barrier energies around the Coulomb barrier height.

This research aims to achieve the surface diffuseness parameters of inter-nucleus potential for the systems $^{34,32}$S + $^{208}$Pb by using large-angle quasi-elastic scattering at deep sub-barrier energies close to the Coulomb barrier height and the single-channels and coupled-channels calculations were Conducted by using CQEL program which includes all orders of coupling and it is considered the latest version of computer code CCFULL [21]. The best fitted values of the diffuseness parameters in comparison with the experimental data have been obtained through the chi square method $\chi^2$ [21].

2. Theory

The nucleus-nucleus potential is consist from two parts [5] nuclear part $V_N$, which can be described well and fairly reasonable by the
Woods-Saxon (WS) form which is given by [10]:

\[
V_N(r) = -\frac{V_0}{1 + \exp]\left[\frac{r - R_0}{a}\right]. (1)
\]

where \( R_0 \) is a radius parameter of the system, \( v_0, a \) and \( r_0 \) represent the potential depth, surface diffuseness parameter, and radius parameter, respectively, whilst \( r \) refers to the center-of-mass distance between the target nucleus of mass number \( A_T \) and the projectile nucleus of mass number \( A_p \) [26].

From another side, Coulomb part \( V_c \) between two spherical nuclei with regular charge density distributions and when they do not interfere is given by [10]:

\[
V_c(r) = \frac{Z_pZ_Te^2}{r}. (2)
\]

\[
H(\vec{r}, \zeta) = -\frac{h^2}{2\mu} \nabla^2 + V(r) + H_0(\zeta) + V_{coup}(\vec{r}, \zeta). (4)
\]

where \( r \) refers to the center of mass distance between the colliding nuclei, \( \mu \) is the reduced mass of the system while \( V(r) \) is the naked potential in the absence of the coupling where \( V(r) = V_N(r) + V_c(r) \), \( H_0(\zeta) \) represents the Hamiltonian for the intrinsic motion, \( V_{coup}(\vec{r}, \zeta) \) is the mentioned coupling [4]. The Schrödinger equation for the total wave function would be given by [4]:

\[
(\frac{h^2}{2\mu} \nabla^2 + V(r) + H_0(\zeta) + V_{coup}(\vec{r}, \zeta))\psi(\vec{r}, \zeta) = E\psi(\vec{r}, \zeta). (5)
\]

The internal degree of freedom \( \zeta \) principally has a limited spin. We can write the coupling Hamiltonian in complications as [4]:

\[
V_{coup}(\vec{r}, \zeta) = \sum_{\lambda>0,\mu} f_{\lambda}(r) Y_{\lambda\mu}(\vec{r}) \cdot T_{\lambda\mu}(\zeta). (6)
\]

\( Y_{\lambda\mu}(\vec{r}) \) refers to the spherical harmonics and \( T_{\lambda\mu}(\zeta) \) refers to the spherical tensors, which are built from the internal coordinate. The sum is taken over all values of excluding for \( \lambda = 0 \) since it is originally considered in \( V(r) \). The expansion basis for the wave function in equation (5) for a fixed total angular momentum \( J \) and its \( z \)-component \( M \) is defined as [4]:

\[
\langle \vec{r} \xi | (nl)JM \rangle = \sum_{m_l} \sum_{m_I} \psi_{nlm_l}(\vec{r}) \varphi_{nlm_I}(\zeta) \psi_{nlm_IJ}JM (\zeta) (7)
\]

where \( l \) refers to the orbital, \( I \) represents the internal angular momenta, and represents the wave function for the internal motion which fulfills [4].
The total wave function $\psi(r, \xi)$ has been expanded with this basis as [4]:

$$\psi(r, \xi) = \sum_{n, l, l'} \frac{u_{nll}^J(r)}{r} \langle \hat{\xi} | (nl)JM \rangle.$$ \hspace{1cm} (9)

The Schrödinger equation (equation (2)) can then be written as a group of coupled equations for $u_{nll}^J(r)$ [4]:

$$-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + V(r) + \frac{l(l + 1)\hbar^2}{2\mu r^2} - E + \epsilon_n \left[ u_{nll}^J(r) + \sum_{n', l', l'} V_{nll; n'l'l'}^J(r) u_{n'l'l'}^J(r) \right] = 0.$$ \hspace{1cm} (10)

Terms the coupling matrix elements is given by [4]:

$$V_{nll; n'l'l'}^J(r) = \langle \hat{JM} | (nl) | \hat{V}_{\text{coupl}} (r) | (n'l'l')JM \rangle = \sum_{\lambda} (-1)^{l-l'+l'} f_{\lambda}(r) \langle l'|Y_{\lambda}\rangle \langle n'l'||l\rangle \times \sqrt{(2l+1)(2l'+1)} \{l' \lambda l \}.$$ \hspace{1cm} (11)

The reduced matrix elements in equation (11) is defined by [4]:

$$\langle l_m\lambda\mu | l'_l\lambda'\mu' \rangle = \langle l'_l\lambda' \lambda\mu | l_m\lambda \rangle \langle l||Y_{\lambda}||l'\rangle.$$ \hspace{1cm} (12)

Since is freelance of the coefficient $M$, the coefficient has been suppressed as seen in equation (11). The equation (10) is called coupled-channels equations. For heavy-ion fusion interactions, these equations are commonly resolved using the incoming wave boundary conditions [4]

$$u_{nll}^J(r) \sim T_{nll}^J \exp \left( -1 \int_{r_{\text{abs}}}^{r} k_{nll}(\tilde{r})d\tilde{r} \right), r \leq r_{\text{abs}}.$$ \hspace{1cm} (13)

$$\rightarrow \frac{i}{2} \left( H_l^{(-)}(k_{nlr}) \delta_{n,n_l} \delta_{l,l_i} \delta_{l',l_i} + \frac{\sqrt{k_{nl}}}{k_{nlr}} S_{l_i}^I H_l^{(+)}(k_{nlr}) \right), r \rightarrow \infty$$

$$k_{nlr} = \sqrt{2\mu(E - \epsilon_{nl})/\hbar^2}, \quad k_{nl} = k = \sqrt{2\mu E/\hbar^2}$$

The local wave number is defined as [4]:

$$k_{nll}(r) = \frac{\sqrt{2\mu}}{\hbar^2} \left( E - \epsilon_{nl} - \frac{l(l + 1)\hbar^2}{2\mu r^2} - V(r) - V_{nll; n'l'l'}^J(r) \right).$$ \hspace{1cm} (15)
Once we obtained the transmission coefficients, the penetrability during the Coulomb barrier is given by:

\[ P_{tii}(E) = \sum_{n,l,l} \frac{k_{nl}(r_{abs})}{k} |T_{nll}|^2 \]  

(16)

is the wave number for the entrance channel. The fusion cross section for unpolarized target is given by:

\[ \sigma_{fus}(E) = \frac{\pi}{k^2} \sum_{J} \frac{2J+1}{2l+1} P_{tii}(E) \]  

(17)

When the initial intrinsic spin = 0, then the initial angular momentum = J, with the coefficients and are suppressed in the penetrability, equation (17) then reads [4]:

\[ \sigma_{fus}(E) = \frac{\pi}{k^2} \sum_{J} 2J + 1 \sigma_{fus}(E) \]  

(18)

\[ f_{ll}^J(\theta, E) = i \sum_{l} \sqrt{\frac{\pi}{k k_{nl}}} i^{l-l} e^{i[\sigma_{l}(E) + \sigma_{c}(E - \epsilon_{nl})]} \sqrt{2J + 1} Y_{l0}(\theta) \] 

(19)

\[ (S_{l1}^J - \delta_{l1} \delta_{l2}) + f_{c}(\theta, E) \delta_{l1} \delta_{l2} \] 

\[ \sigma_{l} \] is the Coulomb phase shift which is given by [4]:

\[ \sigma_{l} = |\Gamma(l + 1 + i\eta)| \]  

(20)

\[ f_{c}(\theta, E) = \frac{\eta}{2k \sin^2(\theta/2)} e^{-i\eta \ln \left( \sin^2 \left( \frac{\theta}{2} \right) \right) + 2i\sigma_{0}(E)} \]  

(21)

\[ \eta \] is the Summerfield parameter, which is given by, we can be evaluated the differential cross section by using equation (19) [4]  

\[ \frac{d\sigma_{qel}(\theta, E)}{d\Omega} = \sum_{l} \frac{k_{nl}}{k} |f_{ll}^J(\theta, E)|^2 \]  

(22)

\[ \frac{d\sigma_{qel}(\theta, E)}{d\Omega} = |f_{c}(\theta, E)|^2 = \frac{\eta^2}{4k^2} \csc^4 \left( \frac{\theta}{2} \right) \]  

(23)

where \( P'(E) \) is the penetrability which is affected now by the channel couplings. Unlike to the calculation of fusion cross sections, the calculation of quasi-elastic cross sections usually requires a large value of angular momentum so as to obtain converged results. The potential pocket at \( (r = r_{abs}) \) becomes superficiality or even disappears for such large angular momentum. Hence, the incoming flux in equation (13) cannot be correctly identified. Therefore, the quasi-elastic problem commonly performs the regular boundary conditions at the origin rather than using the incoming wave boundary conditions. When using the regular boundary conditions, a complex potential \( V_{N}(r) = V_{N0}(r) + iv_{N}(r) \), is needed to simulate the fusion reaction. Once the nuclear S-matrix in equation (11) is obtained, the scattering amplitude can then be calculated as [4]:

\[ f_{ll}^J(\theta, E) = i \sum_{l} \sqrt{\frac{\pi}{k k_{nl}}} i^{l-l} e^{i[\sigma_{l}(E) + \sigma_{c}(E - \epsilon_{nl})]} \sqrt{2J + 1} Y_{l0}(\theta) \] 

(19)

\[ (S_{l1}^J - \delta_{l1} \delta_{l2}) + f_{c}(\theta, E) \delta_{l1} \delta_{l2} \] 

\[ \sigma_{l} \] is the Coulomb phase shift which is given by [4]:

\[ \sigma_{l} = |\Gamma(l + 1 + i\eta)| \]  

(20)

\[ f_{c}(\theta, E) = \frac{\eta}{2k \sin^2(\theta/2)} e^{-i\eta \ln \left( \sin^2 \left( \frac{\theta}{2} \right) \right) + 2i\sigma_{0}(E)} \]  

(21)

\[ \eta \] is the Summerfield parameter, which is given by, we can be evaluated the Rutherford cross section by using equation (21) [4]  

\[ \frac{d\sigma_{qel}(\theta, E)}{d\Omega} = |f_{c}(\theta, E)|^2 = \frac{\eta^2}{4k^2} \csc^4 \left( \frac{\theta}{2} \right) \]  

(23)
3. Procedure
The single-channel and coupled-channels calculations have been carried out using CQEL program, which is considered the latest version of computer code CCFULL [21]. This code solves the Schrödinger equation and the coupled equations exactly [33]. The chi square method $\chi^2$ was considered normalization factor between the theoretical calculation and the experimental data to avoid systematic errors in the present work where the data with $d\sigma_{qs}/d\sigma_R > 1$ were excluded from the fitting proceedings [4, 12]. This calculations were made using a WS form for the nuclear potential, which consists of real and an imaginary component [4, 12]. The values supposed for the parameters of the imaginary part ($w = 30$ MeV, $r_w = 1.0$ fm and $a_w = 0.1$ fm) result in trivial strength in the surface region [24]. The imaginary potential was used to account for the rather small internal absorption from barrier penetration [12]. The imaginary part of the potential remained inside the Coulomb barrier, the results were insensitive to variations of the imaginary potential parameters [4, 12]. The Woods-Saxon (WS). The parameters of the real potential were researched to get the best fit to the experimental data, so it were reproduced for all interactions [4, 12]. The Woods-Saxon (WS). The radius parameter $r_0$ is taken to be 1.2 fm, while the values of potential depth $V_0$ depended on the diffuseness parameter are taken to be 62.5 MeV and 80.5 MeV for the $^{32\,16}S +^{208\,82}Pb$ and $^{32\,16}S +^{208\,82}Pb$ systems, respectively. The radius of the target was taken as $R_T = r_T A^{1/3}$ such that $r_T = 1.16$ fm while for the projectile $R_p = r_p A^{1/3}$ so $r_p = 1.22$ fm. The calculations are performed at scattering angle of $\theta_{lab} = 170^\circ$ for the $^{32\,16}S +^{208\,82}Pb$ system, while $\theta_{lab} = 159^\circ$ for the $^{32\,16}S +^{208\,82}Pb$ system [34, 37]. The experimental data of the quasi-elastic cross sections at deep sub-barrier energies for all systems were taken from the Ref. [36, 37]. We find that the deep sub-barrier region can be defined in this way corresponds to the region where $d\sigma_{qs}/d\sigma_R \geq 0.95$ for $^{32\,16}S +^{208\,82}Pb$ reaction, $d\sigma_{qs}/d\sigma_R \geq 0.93$ for $^{32\,16}S +^{208\,82}Pb$ reaction. We analysis and plot the calculated ratio of the quasi-elastic to the Rutherford cross sections as functions of the center of mass energies, in order to make sure that the calculations are properly consistent according to the available experimental data [24].

4. Results
4.1. The $^{32\,16}S +^{208\,82}Pb$ reaction
This reaction involve spherical nuclei for both projectile $^{34,32\,16}S$ and target $^{208\,82}Pb$ [15]. The characteristics of the single-quadruple phonon excitation for each nucleus are shown in the Table (1), where $\beta$, $\hbar \omega$, $J$, $\pi$, and $\lambda$ are the deformation parameter of the phonon state, excitation energy, angular momentum, parity and vibration mode respectively. [31]

<table>
<thead>
<tr>
<th>Spherical Nuclear</th>
<th>$\beta_0$</th>
<th>$\hbar \omega$ (MeV)</th>
<th>$J^\pi$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{32,16}S$</td>
<td>0.312</td>
<td>2.2303</td>
<td>2$^+$</td>
<td>2</td>
</tr>
<tr>
<td>$^{208,82}Pb$</td>
<td>0.0553</td>
<td>4.0854</td>
<td>2$^+$</td>
<td>2</td>
</tr>
<tr>
<td>$^{34,16}S$</td>
<td>0.252</td>
<td>2.1276</td>
<td>2$^+$</td>
<td>2</td>
</tr>
</tbody>
</table>

Table (1): The characteristics of the single-quadruple phonon excitation for the nuclei.
In the $^{32}{_{16}}S + ^{208}{_{82}}Pb$ system, the diffuseness parameter have been discussed in four states, in the first state we considered the projectile $^{32}{_{16}}S$ as well as target $^{208}{_{82}}Pb$ as inert nuclei (SC), while in the second state we considered the target nucleus $^{208}{_{82}}Pb$ is vibrational coupling with deformation parameter $\beta_0= 0.0553$ to the state $2^+(4.0854 \text{ MeV})$, while the projectile nucleus $^{32}{_{16}}S$ is inert, the third state, we assumed that the projectile nucleus $^{32}{_{16}}S$ is vibrational coupling to the state $2^+$ with deformation parameter $\beta_0= 0.312 (2.2303 \text{ MeV})$, while the target $^{208}{_{82}}Pb$ is inert, in the last state we assumed that projectile $^{32}{_{16}}S$ as well as target $^{208}{_{82}}Pb$ nuclei are vibrational coupling to the state $2^+$. We used single-quadruple phonon excitation for the projectile and target nuclei which were vibrational excited. The values of the diffuseness parameters (a) have been obtained from SC and CC analysis, as well as others parameters of WS potential (radius $r_0$ and depth potential $v_0$) and the values of $\chi^2$ fitting between experimental and theoretical data for the $^{32}{_{16}}S + ^{208}{_{82}}Pb$ reaction were shown in Table (2).

Table (2): parameters of WS potential $a$, $r_0$ and $v_0$ and values of $\chi^2$ fitting between experimental and theoretical data for different types reactions when the excited nuclei at vibrational excitation state with single-quadruple phonon.

<table>
<thead>
<tr>
<th>Type of reaction</th>
<th>a (fm)</th>
<th>r0 (fm)</th>
<th>V0 (MeV)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC (Inert + Inert)</td>
<td>0.65</td>
<td>1.2</td>
<td>62.5</td>
<td>0.178</td>
</tr>
<tr>
<td>CC (Inert + Vib.)</td>
<td>0.63</td>
<td>1.2</td>
<td>62.5</td>
<td>0.120</td>
</tr>
<tr>
<td>CC (Vib. + Inert)</td>
<td>0.62</td>
<td>1.2</td>
<td>62.5</td>
<td>0.126</td>
</tr>
<tr>
<td>CC (Vib. + Vib.)</td>
<td>0.61</td>
<td>1.2</td>
<td>62.5</td>
<td>0.112</td>
</tr>
</tbody>
</table>

By observing the results in Table (2), we find that the better suitable value diffuseness parameter which have obtained from SC analysis (where the projectile $^{32}{_{16}}S$ and target $^{208}{_{82}}Pb$ nuclei are inert) is $0.65 \text{ fm}$ with $\chi^2=0.178$, this result considered very near for standard value $a = 0.63 \text{ fm}$, and represented by the hard line in Fig.(1) (a), while the dashed line represents the single-channel accounts with the diffuseness parameter is $0.55 \text{ fm}$ was drawn for the comparison.

The better suitable value of the diffuseness parameter which have obtained from CC analysis (where we assumed that the projectile $^{32}{_{16}}S$ as inert with vibrational coupling for target $^{208}{_{82}}Pb$ nucleus) is $0.63 \text{ fm}$ with $\chi^2=0.120$, this result considered fully compatible with the standard value $0.63 \text{ fm}$, this is illustrated clearly through preview the hard line in Fig.(1) (b), The dot-dashed line in Fig. (1) (b) represents the result which obtained from CC analysis (where we assumed that the target as inert with vibrational coupling for projectile nucleus) with diffuseness parameter is $0.62 \text{ fm}$ and $\chi^2=0.126$, the dashed line in Fig.(1) (b)
Fig. (1): Comparison of single-channel and different types of coupled-channels accounts with experimental data [15] (Referred to as points with error bars) for the system. In the upper panel (a) the hard and dashed lines represent the results of SC analysis at $a = 0.65$ fm (represents the better suitable value of diffuseness parameter) and $a = 0.55$ fm respectively, while the hard, dashed and dot-dashed lines in the lower panel (b) represent the results of CC analysis at $a=0.63$ fm, $a=0.62$ fm and $a=0.61$ fm and $a=0.62$ fm respectively.
represents the result which got from CC analysis with collective vibrational excitations of the colliding nuclei (where the projectile $^{32}_{16}S$ and target nuclei are vibrational coupling to the state $2^+$) with diffuseness parameter is 0.61 fm and $\chi^2=0.112$. The hard lines in Fig.(2) shows, the $d\sigma_{qel}/d\sigma_R$ at The best fitted diffuseness parameter is 0.63 fm, with $\chi^2=0.120$ using a coupled-channel calculation at deep sub-barrier energies. In this reaction, we assumed that projectile $^{32}_{16}S$ is inert whilst the target $^{208}_{82}Pb$ is vibrational coupling to the state $2^+$. The dashed line in Fig.(2) shows the better suitable value of the diffuseness parameter for the $^{32}_{16}S+^{208}_{82}Pb$ reaction got from SC account is 0.65 fm, with $\chi^2=0.178$, we assumed that the projectile and target as inert nuclei.

In the $^{34}_{16}S+^{208}_{82}Pb$ system, the diffuseness parameter have been discussed in four states, in the first state we considered the projectile $^{34}_{16}S$ as well as target $^{208}_{82}Pb$ as inert nuclei, while in the second state we considered target nucleus $^{208}_{82}Pb$ is vibrational coupling with deformation parameter $\beta_0=0.0553$ to the state $2^+(4.0854$ MeV), while the projectile nucleus $^{34}_{16}S$ is inert, as to for the third state we assumed that the projectile nucleus $^{34}_{16}S$ is vibrational coupling to the state $2^+$ with deformation parameter $\beta_0=0.252(2.1276$ MeV), while the target $^{208}_{82}Pb$ is inert, in the last way we assumed that projectile $^{34}_{16}S$ as well as target $^{208}_{82}Pb$ nuclei are vibrational coupling to the state $2^+$.

We used single-quadruple phonon excitation for the projectile and target nuclei which were vibrational excited. The values of the diffuseness parameters have been obtained from SC and CC analysis, as well as others parameters of WS potential (radius $r_0$ and depth potential $V_0$) and the values of $\chi^2$ fitting between experimental and theoretical data for the $^{34}_{16}S+^{208}_{82}Pb$ reaction were shown in Table (3).

By observing the results in Table (3), we find that the better suitable value of the diffuseness parameter which have obtained from SC analysis (where the projectile $^{32}_{16}S$ and target $^{208}_{82}Pb$ nuclei are inert) is 0.64 fm with $\chi^2=0.557$, this result considered very near to the accepted value of $a=0.63$ fm, and represented by the hard line in Fig.(3) (a), the dashed and dotted lines in Fig. (3) (a) represented the SC analysis with values of diffuseness parameter are 0.66 fm and 0.6 fm respectively, which were drown for the comparison.

The better suitable value of the diffuseness parameter which have obtained from CC analysis (where we assumed that the projectile $^{34}_{16}S$ as inert

### Table (3): parameters of WS potential $a$, $r_0$ and $V_0$ and values of $\chi^2$ fitting between experimental and theoretical data for different types reactions when the excited nuclei at vibrational excitation state with single-quadruple phonon.

<table>
<thead>
<tr>
<th>Type of reaction</th>
<th>$a$ (fm)</th>
<th>$r_0$ (fm)</th>
<th>$V_0$ (MeV)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC (Inert + Inert)</td>
<td>0.64</td>
<td>1.2</td>
<td>94</td>
<td>0.557</td>
</tr>
<tr>
<td>CC (Inert + Vib.)</td>
<td>0.63</td>
<td>1.2</td>
<td>94</td>
<td>0.499</td>
</tr>
<tr>
<td>CC (Vib. + Inert)</td>
<td>0.62</td>
<td>1.2</td>
<td>94</td>
<td>0.523</td>
</tr>
<tr>
<td>CC (Vib. + Vib.)</td>
<td>0.62</td>
<td>1.2</td>
<td>94</td>
<td>0.560</td>
</tr>
</tbody>
</table>
Fig.(2): Comparison of single and coupled-channels accounts for the better suitable value of the diffuseness parameter with experimental data [15] (Referred to as points with error bars) for the system. The hard line represents the results got from a coupled-channel analysis at $a = 0.63$ fm, while the dashed line represents the single-channel analysis at $a = 0.65$ fm.

with vibrational coupling for target $^{208}\text{Pb}$ nucleus) is $0.63$ fm with $\chi^2 = 0.499$, this result considered fully compatible with the standard value $0.63$ fm, this is illustrated clearly through preview the hard line in Fig.(3) (b). The dashed line in Fig. (3) (b) represents the result which obtained from CC analysis (where we assumed that the target as inert with vibrational coupling for projectile nucleus with diffuseness parameter $a = 0.62$ fm and $\chi^2 = 0.523$, the dashed line in Fig.(3) (a) represents the result which got from CC analysis with collective vibrational excitations of the
Fig. (3): Comparison of single-channel and different types of coupled-channels accounts with experimental data [15] (Referred to as points with error bars) for the system. The hard, dashed and dotted lines in the upper panel (a) represent the results of SC analysis at $a=0.64$ fm, $a=0.66$ fm and $a=0.6$ fm respectively while the hard, dashed and dotted lines in the lower panel (b) represent the results of CC analysis at $a=0.63$ fm, $a=0.62$ fm and $a=0.62$ fm respectively.
Fig.(4): Comparison of single and coupled-channels accounts for the better suitable value of the diffuseness parameter with experimental data [15] (Referred to as points with error bars) for the system. The hard line represents the results got from a coupled-channel analysis at $a = 0.63$ fm, while the dotted line represents the single-channel analysis at $a = 0.64$ fm.

Colliding nuclei (where the projectile $^{32}$S and target $^{208}$Pb nuclei are vibrational coupling together to the state $2^+$) with diffuseness parameter is 0.62 fm and $\chi^2=0.560$.

We can comparison between the better suitable value of the diffuseness parameter which have obtained from SC and CC analysis in Fig.(3) (c), such that the hard line in Fig.(3) (c) represents the CC analysis (with inert Projectile and vibrational target) at diffuseness parameter...
is 0.63 fm with $\chi^2 = 0.499$ was drawn for the comparison with dotted line which is represented the SC analysis at diffuseness parameter is 0.64 fm with $\chi^2 = 0.557$.

Fig. (4) (a) shows property of the nuclear potential $V_N$ at the surface region as a function of the distance $r$ between the center of mass of the projectile and the target for the $^{32}\text{S} + ^{208}\text{Pb}$ system, where the largest diffuseness parameter $a=0.65$ fm (represents by the dashed line) which is resulted from SC analyses makes the nuclear potential to become more spread out comparison with the accepted value (represents by the hard line), while the Fig. (4) (b) clears characteristic of the nuclear potential $V_N$ at the surface region as a function of the distance $r$ between the projectile and the target for the $^{34}\text{S} + ^{208}\text{Pb}$ system, where the largest diffuseness parameter is 0.64 fm (represents by the dashed line) compared to diffuseness parameter 0.63 fm (represents by the hard line) which were obtained from single-channel and coupled channel analyses respectively, makes too the nuclear potential to become more spread out [31].

The property of the nuclear potential $V_N$ at the surface region as a function of the distance between the center of mass of the projectile and the target are shown in Fig. (5), where in the upper panel (a) the best fitted value of the diffuseness parameter which have obtained from CC analysis $a=0.63$ fm (represents by the solid line), the dashed line represents the better suitable value of the diffuseness parameter which have obtained from SC analysis at $a=0.65$ fm for the system $^{32}\text{S} + ^{208}\text{Pb}$, while the sold line in the lower panel (b) represents the better suitable value of the diffuseness parameter which have obtained from CC analysis at $a=0.63$ fm, the dashed line represents the better suitable value of the diffuseness parameter which have obtained from SC analysis $a=0.64$ fm for the system $^{32}\text{S} + ^{208}\text{Pb}$.

5. Conclusions

Through micro methodology analyzes of the results, we found that the method of large-angle quasi-elastic scattering at deep sub-barrier energies close to the Coulomb barrier height is ideal tool for studying the surface property of Inter- nucleus potential for the spherical systems referred to in this research. Single-channel analyzes fits to experimental data gives diffuseness parameters 0.65 fm and 0.64 fm for the systems $^{32}\text{S} + ^{208}\text{Pb}$ and $^{34}\text{S} + ^{208}\text{Pb}$ respectively, does not differ much from the best fitted value of the diffuseness parameter which have obtained from CC analysis (with inert projectile and vibrational target) $a=0.63$ fm which are in complete agreement with the standard value $a=0.63$ fm. All coupling channels accounts gave values close to the standard value of the diffuseness parameter.

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Fig.(5): Show the property of the nuclear potential $V_N$ (MeV) at the surface region as a function of the distance $r$ (fm) between the center of mass of the projectile and the target. The upper panel (a) for system and the lower panel (b) for the system.
References